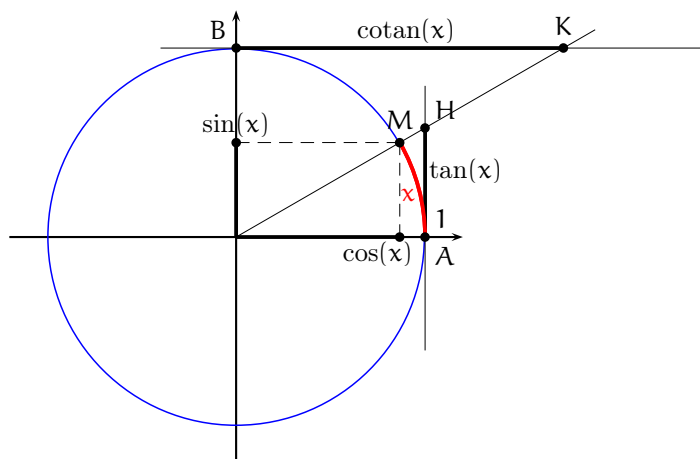


Formulaire de trigonométrie circulaire



$\cos(x)$ = abscisse de M
 $\sin(x)$ = ordonnée de M
 $\tan(x) = \overline{AH}$
 $\cotan(x) = \overline{BK}$
 $e^{ix} = z_M$

Pour $x \notin \frac{\pi}{2} + \pi\mathbb{Z}$, $\tan(x) = \frac{\sin(x)}{\cos(x)}$ et pour $x \notin \pi\mathbb{Z}$, $\cotan(x) = \frac{\cos(x)}{\sin(x)}$. Enfin pour $x \notin \frac{\pi}{2}\mathbb{Z}$, $\cotan(x) = \frac{1}{\tan(x)}$.

Valeurs usuelles.

x en °	0	30	45	60	90
x en rd	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(x)$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(x)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\cotan(x)$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

$$\forall x \in \mathbb{R}, \cos^2 x + \sin^2 x = 1$$

$$\forall x \notin \frac{\pi}{2} + \pi\mathbb{Z}, 1 + \tan^2 x = \frac{1}{\cos^2 x}$$

$$\forall x \notin \pi\mathbb{Z}, 1 + \cotan^2 x = \frac{1}{\sin^2 x}$$

addition d'un tour

$$\begin{aligned} \cos(x + 2\pi) &= \cos x \\ \sin(x + 2\pi) &= \sin x \\ \tan(x + 2\pi) &= \tan x \\ \cotan(x + 2\pi) &= \cotan x \end{aligned}$$

angle complémentaire

$$\begin{aligned} \cos\left(\frac{\pi}{2} - x\right) &= \sin x \\ \sin\left(\frac{\pi}{2} - x\right) &= \cos x \\ \tan\left(\frac{\pi}{2} - x\right) &= \cotan x \\ \cotan\left(\frac{\pi}{2} - x\right) &= \tan x \end{aligned}$$

addition d'un demi-tour

$$\begin{aligned} \cos(x + \pi) &= -\cos x \\ \sin(x + \pi) &= -\sin x \\ \tan(x + \pi) &= \tan x \\ \cotan(x + \pi) &= \cotan x \end{aligned}$$

quart de tour direct

$$\begin{aligned} \cos\left(x + \frac{\pi}{2}\right) &= -\sin x \\ \sin\left(x + \frac{\pi}{2}\right) &= \cos x \\ \tan\left(x + \frac{\pi}{2}\right) &= -\cotan x \\ \cotan\left(x + \frac{\pi}{2}\right) &= -\tan x \end{aligned}$$

angle opposé

$$\begin{aligned} \cos(-x) &= \cos x \\ \sin(-x) &= -\sin x \\ \tan(-x) &= -\tan x \\ \cotan(-x) &= -\cotan x \end{aligned}$$

quart de tour indirect

$$\begin{aligned} \cos\left(x - \frac{\pi}{2}\right) &= \sin x \\ \sin\left(x - \frac{\pi}{2}\right) &= -\cos x \\ \tan\left(x - \frac{\pi}{2}\right) &= -\cotan x \\ \cotan\left(x - \frac{\pi}{2}\right) &= -\tan x \end{aligned}$$

angle supplémentaire

$$\begin{aligned} \cos(\pi - x) &= -\cos x \\ \sin(\pi - x) &= \sin x \\ \tan(\pi - x) &= -\tan x \\ \cotan(\pi - x) &= -\cotan x \end{aligned}$$

Formules d'addition

$$\begin{aligned}\cos(a+b) &= \cos a \cos b - \sin a \sin b \\ \cos(a-b) &= \cos a \cos b + \sin a \sin b \\ \sin(a+b) &= \sin a \cos b + \sin b \cos a \\ \sin(a-b) &= \sin a \cos b - \sin b \cos a\end{aligned}$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

Formules de duplication

$$\begin{aligned}\cos(2a) &= \cos^2 a - \sin^2 a \\ &= 2 \cos^2 a - 1 \\ &= 1 - 2 \sin^2 a \\ \sin(2a) &= 2 \sin a \cos a\end{aligned}$$

$$\tan(2a) = \frac{2 \tan a}{1 - \tan^2 a}$$

Formules de linéarisation

$$\cos a \cos b = \frac{1}{2}(\cos(a-b) + \cos(a+b))$$

$$\sin a \sin b = \frac{1}{2}(\cos(a-b) - \cos(a+b))$$

$$\sin a \cos b = \frac{1}{2}(\sin(a+b) + \sin(a-b))$$

$$\cos^2 a = \frac{1 + \cos(2a)}{2}$$

$$\sin^2 a = \frac{1 - \cos(2a)}{2}$$

Formules de factorisation

$$\cos p + \cos q = 2 \cos \frac{p+q}{2} \cos \frac{p-q}{2}$$

$$\cos p - \cos q = -2 \sin \frac{p+q}{2} \sin \frac{p-q}{2}$$

$$\sin p + \sin q = 2 \sin \frac{p+q}{2} \cos \frac{p-q}{2}$$

$$\sin p - \sin q = 2 \sin \frac{p-q}{2} \cos \frac{p+q}{2}$$

cos x, sin x et tan x en fonction de t=tan(x/2)

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$\sin x = \frac{2t}{1+t^2}$$

$$\tan x = \frac{2t}{1-t^2}$$

Divers

$$1 + \cos x = 2 \cos^2 \frac{x}{2}$$

$$1 - \cos x = 2 \sin^2 \frac{x}{2}$$

$$\cos(3x) = 4 \cos^3 x - 3 \cos x$$

$$\sin(3x) = 3 \sin x - 4 \sin^3 x$$

Résolution d'équations

$$\cos x = \cos a \Leftrightarrow$$

$$\exists k \in \mathbb{Z} / x = a + 2k\pi$$

ou

$$\exists k \in \mathbb{Z} / x = -a + 2k\pi$$

$$\sin x = \sin a \Leftrightarrow$$

$$\exists k \in \mathbb{Z} / x = a + 2k\pi$$

ou

$$\exists k \in \mathbb{Z} / x = \pi - a + 2k\pi$$

$$\tan x = \tan a \Leftrightarrow$$

$$\exists k \in \mathbb{Z} / x = a + k\pi$$

Exponentielle complexe

$$\forall x \in \mathbb{R}, e^{ix} = \cos x + i \sin x.$$

Valeurs usuelles

$$e^0 = 1, e^{i\pi/2} = i, e^{i\pi} = -1, e^{-i\pi/2} = -i, e^{2i\pi/3} = j = -\frac{1}{2} + i\frac{\sqrt{3}}{2}, \sqrt{2}e^{i\pi/4} = 1 + i.$$

Propriétés algébriques

$$\forall x \in \mathbb{R}, |e^{ix}| = 1.$$

$$\forall (x, y) \in \mathbb{R}^2, e^{ix} \times e^{iy} = e^{i(x+y)}, \frac{e^{ix}}{e^{iy}} = e^{i(x-y)}, \frac{1}{e^{ix}} = e^{-ix} = \overline{e^{ix}}$$

Formules d'EULER

$$\forall x \in \mathbb{R}, \cos x = \frac{e^{ix} + e^{-ix}}{2} \text{ et } e^{ix} + e^{-ix} = 2 \cos x.$$

$$\forall x \in \mathbb{R}, \sin x = \frac{e^{ix} - e^{-ix}}{2i} \text{ et } e^{ix} - e^{-ix} = 2i \sin x.$$

Formule de MOIVRE

$$\forall x \in \mathbb{R}, \forall n \in \mathbb{Z}, (e^{ix})^n = e^{inx}.$$